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# **SAMPLING STRATEGY OF FREE-FORM SURFACE INSPECTION USING CONTACT MEASUREMENT**

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# **Abstract**

Inspection of a free-form surface using Coordinate Measuring machine (CMM) can be challenging, particularly if the surface has a lot of curvature. CMM sampling strategies are necessary for companies to examine free-form surfaces quickly and affordably. The inspection cost in coordinate metrology with a CMM is directly related to sample size. A sound sampling plan lowers the inspection cost and sample size. This paper proposes a partial-novel adaptive method of the Modified Translation Propagation Latin Hypercube Design (MTPLHD) algorithm combined with Gaussian Curvature Patch (GP). The robustness and effectiveness of the adaptive method are compared with the most widely used sampling strategy, equiparametric sampling strategy.

*Keywords:* Adaptive Sampling Strategy, CMM, Free-form Surface, Hypercube Design Latin.

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Free-form surfaces are surfaces do not have an axis of symmetry. These surfaces and have wide application in industries such as aerospace, automotive, consumer goods die and mould making. The contemporary manufacturing industries produces high-variety complex surfaces, tight tolerance and high-quality products [1]. Production industry to be competitive and efficient the quality assurance of complex surface is imperative. Due to high demand of free-form products, the role of CMM to inspect these surfaces become important [2]. However, there is a trade-off relation between inspection cost and sample size for specified tolerance [3]. Generally, measurement in coordinate metrology involves contact and non-contact method[4]. Measurement speed is much lower in contact probe than the non-contact method as the latter can acquire thousands of data points over a large spatial range at a time but with low resolution and costly [5]. The CMM with touch triggered probes (contact) technique involves gathering the coordinate values of the measured points from the free-form surface. As a result, a set of discrete data point in the form of x, y, z obtained [6]. In coordinate metrology, the sample size, and there location are important factors in evaluation of surface deviation [7]. Carefully selection of samples location and size can provide accurate free-form surface inspection. The measured surface is compared to the virtual model by creating a common coordinate reference. Actual measured data are fitted onto a virtual surface and compared for a user defines tolerance [8]. The design surface (nominal) model is constructed in the design coordinate system (DCS) and the actual surface is manufactured in the measurement coordinate system (MCS) [9] [10] [11][12][13].

Generally, sampling strategy is classified into three main categories: blind samplings, adaptive sampling, and manufacturing signature based sampling. Adaptive sampling strategies consider the complexity of the free-form surface and start with a small set of predetermined points, and iteratively add samples until the sampled points meet the stopping criteria (tolerance value). Research works have been done on adaptive sampling strategies to evaluate free-form surface using CMM. Shivakumar Gilbert et.al. [14] Proposed adaptive sampling plans based on Gaussian process models, and sample selection is doe based on predictions obtained from a GP model estimated. Results show that good predictive capability of the strategy in terms of quality of the estimate error and cost of the inspection. Mingrang et.al. [15] Established an adaptive sampling strategy based on the form error model, at the same time they introduced the modification algorithm deviation and uncertainty analysis of the sampling strategy of free-form

surface. Keqing et.al. [16] Suggested adaptive sampling approach of unknown free-form surface, based on advanced path detecting method by integrating one touch trigger probe and two laser probes onto the same axis of CMM and sampling efficiency and accuracy improved significantly. Luca P. and Paul J. [17] introduced a sampling strategy of different scale freeform surface, based on the shape decomposition method. They select the number of the samples according to the complexity of the shape, and recommended that confirmed sampling method achieved better result when surface has abrupt changes. From the above literatures different sampling strategies (sample size and location) provide different measurement results for the same free-form surface [18]. This indicates that measuring a finite number of discrete points on the surface are actually describe by infinite number of samples. Since geometric slope are different at each point, measurement results depend on the sample size and location of these points [19] and still a good sampling strategy is require for inspection of free-form surfaces.

In this paper, sampling strategy of Modified Translation Propagation Latin Hypercube Design algorithm combined to Gaussian Curvature Patch (MTPLHD+GP) is proposed.

#### **2. NURBS Free-form Surface Representation**

The free-form surface  $S(u, v)$  in this paper is NURBS (Non-Uniform Rational B-Spline) surface. The virtual surface is designed using Rhinocreo 5 (student version) as shown in the Figure 1. The mathematical equation of NURBS surface is [20]:

$$
S(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} R_{ij}(u,v) P_{ij}
$$
 (1)

$$
R_{ij}(u,v) = \frac{N_{i,p}(u)N_{j,q}(v)W_{i,j}}{\sum_{k=0}^{n} \sum_{i=0}^{m} N_{k,p}(u)N_{i,q}(v)W_{k,i}}
$$
(2)

Where  $N_{i,p}(u)$ ,  $N_{i,q}(v)$  are base functions along *u* and *v*;  $P_{i,j}$  is control points;  $W_{i,j}$  is the weight of each control points.



**Figure 1**. Free form NURBS surface of model degree 3.

#### **3. Tools Used for Inspection of NURBS Free-form Surfaces**

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# **3.1. Latin hypercube design (LHD)**

LHD is under the category of stratified random sampling technique, which was introduced by McKay and Iman Conover as shown in Figure 2. It is widely used for obtaining a small size design of experiment(DOE) to train the surrogate models, such as Gaussian regression model and Kriging model [21]. The Latin hyper square follows the idea of a Latin square where there is only one sample in each row and each column. The Latin Hypercube generalizes this concept to an arbitrary number of dimensions. In the Latin Hyper Square of a multivariate distribution, a sample size *N* from multiple variables is drawn such that for each variable the sample is marginally maximally stratified [22]. The Translation Propagation Latin Hypercube Design (TPLHD) algorithm was developed by Felipe, A.,Viana [23]. The TPLHD has good space filling property and easy to implement. In this work Translation Propagation Latin Hypercube Design (TPLHD) is used to evaluate complex free-form NURBS surface of different complexity.



**Figure 2.** (a) Surface division into number of blocks (b) Concept of point location in space using Translation Propagation Latin Hypercube Design [23].

# **3.1.1. Modified Translation Propagation Latin Hypercube Design Algorithm (MTPLHD)**

In Modified Translation Propagation Latin Hypercube Design (MTPLHD) sample distribution, there is addition of two *the* sample points on NURBS surfaces. The two samples are added on opposite corner of the surfaces. Therefore, the total sample size will be modified, that is Modified Translation Propagation Hypercube Design (MTPLHD). MTPLHD is constructed based on dimensional variable  $(N_{axis})$  and dimensional number of samples in the design block  $(N_{sh})$ . In design block, the initial samples range from one to five in addition to plus four samples at the corner of the surface. Total sample size  $(N<sub>s</sub>)$  sample in the design spaces divide into a total of number of blocks  $(N_h)$  and number of division  $(N_d)$  as shown on Figure 3. Once the

user fixed the initial number of sample point in the block  $(N_{sb})$  and the total samples size  $N_s$  of the MTPLHD is:

$$
N_b = \frac{N_s}{N_{sb}}\tag{3}
$$

$$
N_{sb} = 2^{Naxis} \tag{4}
$$

The number of division the design surface  $N_d$ 

$$
N_d = (N_b)^{1/Naxis} \tag{5}
$$



**Figure 3**. Concept of point location on the grid design surface for (9+2) using Modified Translation Propagation Latin Hypercube Design (MTPLHD).

#### **3.2. Gaussian curvature patch (GP)**

NURBS model is can be divided into minimum and maximum patch as shown Figure 4 below. The samples are allocated in each patch based on the size of the patch and magnitude of patch curvature. If the first and the second derivative of a point on the surface along x axis is  $P^u$  and  $P^{\mu\nu}$  respectively and  $P^{\nu}$  and  $P^{\nu\nu}$  is along y axis [24]. Equation of Gaussian curvature of a surface in  $R<sup>3</sup>$  can be expressed in the forms of:

$$
K = \frac{LN - M^2}{EG - F^2} \tag{6}
$$

Where:  $L(u, v) = \hat{n}P^{uu}$ ,  $N(u, v) = \hat{n}P^{vv}$ ,  $M(u, v) = \hat{n}P^{uv}$ ,  $E(u, v) = P^u P^u$ ,  $G(u, v) = P^v P^v$ ,  $F(u, v) = P^u P^v$ ,

 $\hat{n}$  = *a* unit normal vector.



**Figure 4**. (a) Mean surface Gaussian curvature distribution (b) Gaussian curvature patch.

# **3.3 Equi-parametric sampling strategy**

In equi-parametric sampling strategy, samples are equally distributes in the *u-v* space of the free-form surface as shown in Figure 5[25]. This sampling strategy is the simple and easily to implement. In this sampling strategy samples are distribute without considering the surface curvature.



**Figure 5**. Equi-parametric sample point distribution.

# **3.4. Substitute surface construction**

BJET Vol. 3, No. 3, August, 2024: 43–57 ©EiT-M, Mekelle University Surface construction is the most important stage in inspection. A poor surface construction causes more error in form deviation analysis. The measured samples are used to construct the substitute geometry of surface. The constructed surface geometry is compared to design model surfaces to assess the conformity. Sample points are fitted to get the desire surface using least square fitting (2.5D best fit plane) based on Delaunay triangulation interpolation principle [26]. In Delaunay triangulation the samples are first projected on the best fitting plane (least squares). The corresponding 2D points are triangulated and the mesh structure is located samples [27]. After the mesh surface is constructed smoothing is applied to approximate the constructed surface. The final constructed surface is used for comaprsion to the virtual model.

#### **3.5. Form deviation calculation and insepection uncertainty analysis**

Prior to the determination of the surface form deviation, it is necessary to fit the measurement data to the nominal design surface. In this paper, the maximum surface deviation is assumed to be 0.5mm. The average form deviation of between surfaces is considering as actual deviation. Since the measured values are discrete and few in number it is not completely desire surface. A good fitted constructed surface can be obtained by converted the surface into dense discrete cloud point. An ideal (nominal) shape of the surface element can be described by the shape function  $N(p)$ , where  $p$  denotes feature variables describing the surface.

$$
A(p) = N(p) + d(p) \tag{7}
$$

$$
d(p) = \hat{n}[A(p) - N(p)] \tag{8}
$$

$$
d_{avg} = \sum_{i=1}^{n} \frac{d(p)}{n} \tag{9}
$$

Where:  $A(p)$  is the actual geometric form of the surface,  $d(p)$  is form deviation,  $d_{avg}$  is average form deviation for repeated measurement actual from nominal,  $n$  is number of measurements,  $\hat{n}$ is unit normal of the data point to the surface.

 $N_s$  output samples are discretized from the surface randomly and  $N(0.005,0.01)$  noise is added to the measurement to find the most convenient surface[28]. An average of 40 repeated measurements using equi-parametric sampling strategy is considered as the true form deviation of the surface. During measurement analysis, the other major source of error which is alignment error also main factor to the uncertainty of the measurement. Mostly, form deviation depends on the measurement uncertainty and alignment error. Thus, a statistical analysis for *n* repetitive measurement is important to reduce measurement error [29]:

$$
s^{2}(d) = \frac{1}{n-1} \sum_{1}^{n} (d_{i} - d_{avg}(x))^{2}
$$
 (10)

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Where  $d_i$  is the global deviation obtained from the adaptively measured data in each measurement, and  $d_{avg}(x)$  is the average of them. Then, the type A uncertainty  $u(d)$  can be calculated as:

$$
u(d) = \sqrt{\frac{s^2(d)}{n}}\tag{11}
$$

#### **4. Methodology of the Experiment**

The virtual complex free-form NURBS of degree three surface was design in Rhino creo4 (Student version 2015). To verify the adaptive method, Aluminium alloy (Al-6061) of surface size 100mm×100mm is used. The prototype is machined on CNC VM machine. The machined surfaces are subjected to measurement using CMM based on the sample distribution of Modified Translation Propagation Latin hypercube design (MTPLHD) combined with Gaussian curvature patch. Initially,  $N_s$  sample points are distributed on NURBS surface. Discrete data samples were collected using CMM of 2mm ball contact stylus and the same form deviation was analysed for defined form tolerance. If the deviation is within the define tolerance (the assumed tolerance in this research is  $\pm 0.5$ mm) then iterative processes is terminated as shown in the flowchart Figure 6. Otherwise the sample size  $N<sub>S</sub>$  is increase based on the GP iteratively. Finally the  $N_s$  output sample size is used to guide the CMM to evaluate the actual NURBS freeform surface.



**Figure 6**. Free-form surface evaluation of workflow.

50

# **5. Experimental Results**

The Free-form surfaces are measured using MITUTOYO CMM: a model of Crysta-plus M544, TP200 probe, style of ball diameter 2 mm as shown in Figure 7(b). CMM environment temperatures was maintain at  $20^0c \pm 1^0c$  CMM operating temperature. The maximum permissible error of the CMM is  $E=(3.5+4.5L/1000)\mu$ m L is work volume of the machine which is L= $\sqrt{x^2 + y^2 + z^2}$  for CMM working volume of *x*=500mm, *y*=400 and *z*=400mm. The models surface is made up of Aluminum alloy (Al-6061) size of 100mmx100mm and machined on 3-axis vertical CNC milling, model of VML800, spindle speed of 2000rpm with accuracy ±0.005mm. Samples were distributed on the surface using the MTPLHD. If the deviation is not within the tolerance limit i.e.  $\pm 0.5$ mm, samples were added iteratively on the surface using GP. Since, the maximum and minimum Gaussian patch has the same curvature, the concept how the sample allocate on each patch is explain below in Figure 7(a). Surface which is divided into different areas, based on the Gaussian Curvature model, A1, A2, and A3, AT (total surface area). The area of each patch from the virtual model was  $A1=1914.34354$  mm<sup>2</sup>,  $A2=1873.449$ mm<sup>2</sup>, A3=1241.28117mm<sup>2</sup>, A4=1035.55478mm<sup>2</sup>, AT=10,000mm<sup>2</sup>. In this model the largest area of the patch is A1≈1914.34354 mm<sup>2</sup>. It was assumed that 10 sample sizes to this patch, the other patch sample size relative to this patch will be, sample of patch2 (10\*1873.45/1914.34≈10), sample of patch3 (10\*1241.28/1914.34≈7), and patch4 (10\*1035.554/1914.34≈6), then the sample is distributed randomly on each patch. After the sample is added to the patch the actual surface of sample distribution is looks like,

*Sample size=18 points (Initially by MTPLHD) +10 points (patch1) +9 points (patch2) + 7 Points (patch 3)* + 6 ponts*(Patch 4)*

*Total sample size=initial sample of MTPLHD + sum of sample of each patches*



**Figure 7**. (a) NURBS Surface patches (b)M544 CMM.



**Figure 8**. Adaptive sampling strategy (a) sample distribution (b) form deviation (c) Tolerance based form deviation (d) alignment distribution deviation.



**Figure 9**. Proposed method average form deviation relative to equi-parametric.



**Figure 10**. Equi-parametric sampling strategy (a) sample distribution (b) form deviation (c) Tolerance based form deviation (d) alignment distribution deviation.



Table 1: Comparison adaptive sampling strategy and equi-parametric form deviation.

Alignment is the important steps to reduce error of the form deviation. Noise error N(0.005, 0.01) deviation was added to the measured row data to get the actual discrete samples location. From the above alignment graph, Figure 8(d) and Figure 10(d) the alignments are almost normal in distribution both for the adaptive method and the equi-parametric sampling strategy. The form deviation distribution and tolerance form deviation distribution Figure 8(c), and 10(c) both sampling strategy have maximum form deviation of 5.064% out of the total form deviation. From the above Figure 9 and Figure 10(b) the form deviation of the adaptive sampling (the proposed method) and the equi-parametric sampling (others method) are equal. Table 1 shows that the equi-parametric samples 97.521 % (118/121) are within  $+/-$  (3  $*$  Std. Dev.) and the same for the adaptive method 100 %  $(52/52)$  is within  $+/- (3 * Std$ . Dev.). Since all the samples are within the tolerance there is no patch out of tolerance.

# **6***.* **Conclusions**

BJET Vol. 3, No. 3, August, 2024: 43–57 ©EiT-M, Mekelle University The proposed partial novel method of adaptive sampling strategy (MTPLHD+GP) use to inspect free-form surface and the same compared to the Equi-parametric sampling strategy for form deviation prediction. The developed strategy reduces the sample size by 57% compared to the Equi-parametric sampling strategy. Measurement CMM CNC mode, time of measurement reduce by 48.5%. The proposed method is effective compared to the equiparametric sampling strategy. The developed strategy is robust since form deviation distribution band between the two methods is 0.00435mm and 0.014mm respectively. Therefore, the proposed method can easily adapt for inspection of complex surfaces at low cost and good accuracy thereby avoid complex calculations.

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#### **References**

- [1] Poniatowska, M., Characteristics of geometric errors determined using discrete measurement data. Archiwum Technologii Maszyn i Automatyzacji 2008; 28( 2), 51-57.
- [2] Nit, P., Huang, S.. On-line error compensation of coordinate measuring machne. International Journal Machine Tools Manufacturing 1995; 35(5), 725-738. doi: 10.1016/0890-6955(95)93041-4.
- [3] Lee, G., Mou, J., Shens, Y.. Sampling strategy design for dimensional measurement of geometric features using coordinate measuring machine. International Journal Machine Tools Manufacturing 1997*;* 37(7), 917-934. doi: 10.1016/S0890-6955(96)00096-X.
- [4] Ainsworth, I., Ristic, M., Brujic, D.. CAD-Based Measurement Path Planning for Free-Form Shapes Using Contact Probes. International Journal Advanced Manufacturing Technololog*y* 2000*,*16(1),23-31. doi: 10.1007/PL00013128.
- [5] Gao, J., Gindy, N., Chen, X.. An automated GD&T inspection system based on noncontact 3D digitization. International Journal of Production Research 2006*,* 44(1),117– 134. doi: 10.1080/09638280500219737.
- [6] Wayne A. F., *Sampling Statistics*. 1 st edition. Wiley. 2009
- [7] Bianca Maria, C., Ester Gutierrez, M., Giovanni, M.. Statistical Sampling Strategies for Geometric Tolerance Inspection by CMM. Economic Quality Control 2008; 23(1). 109- 121. doi: 10.1515/eqc.2008.109.
- [8] Mohamed A., Gadalla E.. Tolerancing of Free Form Surfaces in Geometric Design Tolerancing: Theories, Standards, and Applications; Springer 1998; 387-397.

BJET Vol. 3, No. 3, August, 2024: 43–57 ©EiT-M, Mekelle University [9] Menq, Hong-tzong, Y., Chia-hsiang. A unified least-squares approach to the evaluation of geomp, tric errors using discrete measurement data. Internation Journal Machine Tools Manufacturing 1996*;* 36(12); 1269-1290. doi:10.1016/0890-6955(95)00075-5.

- [10] Se Yun, H., Cheol Ho, R., Jang Hyun, L. Localization between Curved Shell Plate and Its Unfolded Shape in Different Coordinate Systems for Ship-Hull Plate Forming. Mathematical Problems in Engineering 2011; 0-15. doi: 10.1155/2011/257804.
- [11] Huang X., Gu P., Zernicke R. Localization and comparison of two free-form surfaces. Computer-Aided Design 1996*;* 26;1017-1022.doi: 10.1016/0010-4485(96)00011-5.
- [12] Yadong, L., Peihua, G. Free-form surface inspection techniques state of the art review.Computer-Aided Design 2004; 36;1395-1417. doi: 10.1016/j.cad.2004.02.009.
- [13] Yadong, L., Peihua, G.. Inspection of free-form shaped parts. Robotics and Computer*-*Integrated Manufacturing 2005; 21(1); 421–430. doi: 10.1016/j.rcim.2004.11.015.
- [14] S. Raman, R. C. Gilbert, and T. B. Trafalis, "Coordinate metrology for adaptive form verification," Manuf. Lett., 2013, doi: 10.1016/j.mfglet.2013.10.003.
- [15] Mingrang, Y., Yingjie, Z., Yunlong, L., Ding, Z. Adaptive sampling method for inspection planning on CMM for free-form surfaces. International Journal Advanced Manufacturing Technology 2013; *(*1)67; 1967–1975. doi: 10.1007/s00170-012-4623-0.
- [16] Keqing, L., Wen, W., Yaofeng, W., Yanding, W., Zichen, C. An adaptive sampling approach for digitizing unknown free-form surfaces based on advanced path detecting. *Proc. of ELSEVIER B.V.*2013*,* Hangzhou, P.R.China.doi: 10.1016/j.procir.2013.08.034.
- [17] Scotta, Luca, P., Paul, J. A sampling strategy based on B-wavelets decomposition. *Proc. of ELSEVIER CIRP2016*; Huddersfield, England, 29–34.doi: 10.1016/j.procir.2016.01.028.
- [18] Badar , M. A., Raman, S., Pulat P.S., Experimental verification of manufacturing error pattern and its utilization in form tolerance sampling. Int. J. Mach. Tools Manuf.2005; doi: 10.1016/j.ijmachtools.2004.06.017.
- [19] Salah H. R. Ali, "Probing system characteristics in coordinate metrology," Meas. Sci. Rev*.*2010, doi: 10.2478/v10048-010-0023-5.
- [20] Les P., Wayne T., "The NURBS book, 2nd edition, Springer-Verlag. 1997.

BJET Vol. 3, No. 3, August, 2024: 43–57 ©EiT-M, Mekelle University [21] Xiaoping, L., Xueliany, Y., Wei, X., Bin, L., A fast optimal latin hypercube design for

gaussian process regression modeling. 3<sup>rd</sup> International Workshop on Advanced Computational Intelligence, IWACI 2010. doi: 10.1109/IWACI.2010.5585160.

- [22] McBratney, Budiman, M., Alex, B.. A conditioned Latin hypercube method for sampling in the presence of ancillary information. Computers and Geosciences 2006; 32(1); 1378–1388. doi: 10.1016/j.cageo.2005.12.009.
- [23] Felipe, A.,Viana, Gerhard, V., Vladimir, B.. An Algorithm for Fast Optimal Latin Hypercube Design of Experiments. International Journal for Numerical Methods in Engineering 2009; 82(2);135–156. doi: 10.1002/nme.2750.
- [24] T. Banchoff and S. Lovett, Differential geometry of curves and surfaces. Chapman and Hall/CRC; 2nd edition; New Jersey: Prentice Hall 2010.
- [25] Diaa, F. , Hoda, E., Waguih, H.. Automatic sampling for CMM inspection planning of free-form surfaces. International Journal of Production Research 2002; 40(11),2653- 2676. doi: 10.1080/00207540210133435.
- [26] Teng, L., Di, W., Xin, C., Xiaosong, G., Li, L. A Deterministic Sequential Maximin Latin Hypercube Design Method using Successive Local Enumeration for Metamodel-based Optimization. Engineering Optimization2016*,* 48(6); 1019–1036. doi: 10.1080/0305215X.2015.1081518.
- [27] Remondino, F. "From point cloud to surface: the modeling and visualization problem," Int. Arch. Photogramm. Remote Sens*.* Spat. Inf. Sci., 2003.
- [28] Y. F. Li and Z. G. Liu, Method for determining the probing points for efficient measurement and reconstruction of Free-form surfaces. Meas. Sci. Technol*.*, 2003, doi: 10.1088/0957-0233/14/8/313.
- [29] Goitom T. and Rega R., Global form deviation evaluation of free-form surface, using coordinate measuring machine. Int. J. Mech. Prod. Eng. Res. Dev.2017; 525-536. doi: 10.24247/ijmperddec201760.